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DISCRETE ALPHA POWER INVERSE LOMAX DISTRIBUTION WITH APPLICATION OF COVID-19 DATA

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ABSTRACT

This research aims to manage the risks of spreading Corona-Virus over the world, by specifying the optimal statistical modeling to analyze the daily count of new cases of the COVID-19, therefore discrete distributions were needed. A new three-parameter discrete distribution has been improved named as a Discrete Marshall–Olkin Lomax (DAPIL) distribution. Probability mass function and hazard rate are discussed. Point estimation and confidence interval by using maximum likelihood estimation (MLE) for the DAPIL distribution parameters are discussed. A numerical study is done using the daily count of new cases in Australia. Monte Carlo Simulation has been performed to evaluate the restricted sample properties of the proposed distribution.

KEYWORDS: *COVID-19; Hazard Rate; Discrete Distributions; Survival Discretization; Maximum Likelihood Estimation; Confidence Interval*

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INTRODUCTION

In December 2019, Corona-Virus "COVID-19" was started in Wuhan, China. On March 11, 2020, World Health Organization (WHO) described COVID-19 as a pandemic. See Figure 1 and 2. Therefore, countries around the world have been increased their measures trying to decrease the spread rate of the COVID-19.

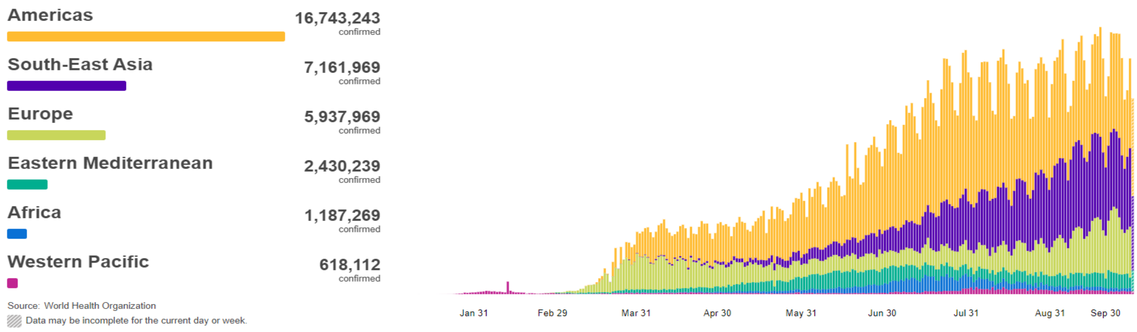

Figure 1: The Situation for the Daily New Cases Over the World by the WHO Region.

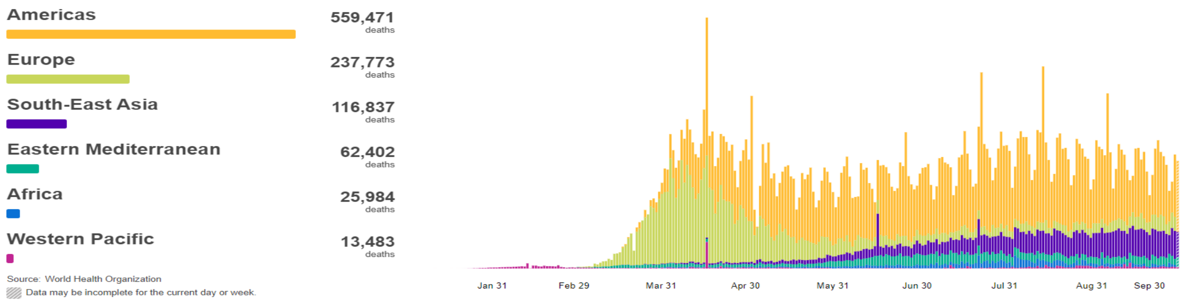

Figure 2: The Situation for the Daily New Deaths Over the World by the WHO Region.

To model daily cases and deaths in the world, a natural discrete Lindley distribution has been introduced by Al-Babtain et al. (2020). A discrete Marshall-Olkin generalized exponential distribution has been introduced by Almetwally et al. (2020) to discuss the daily new cases of Egypt. The study was carried out by Hasab et al. (2020) where they used the Susceptible Infected Recovered (SIR) epidemic dynamics of the COVID-19 pandemic for modeling of the novel Coronavirus epidemic in Egypt. El-Morshedy et al. (2020) studied a new discrete distribution, called discrete generalized Lindley, to analyze the counts of the daily coronavirus cases in Hong Kong and daily new deaths in Iran. An autoregressive time series model based on the two-piece scale mixture normal distribution has been used by Maleki et al. (2020) to forecast the recovered and confirmed COVID-19 cases. The daily new COVID-19 cases in China have been predicted by Nesteruk (2020) and Batista (2020b) by using the mathematical model, called susceptible, infected, and recovered (SIR). Batista (2020a) used the logistic growth regression model is used for the estimation of the final size and its peak time of the coronavirus epidemic.

In inverse Lomax (IL) is originally developed as a lifetime distribution. The IL is member of family of generalized beta distribution. Kleiber and Kotz (2003) showed that IL distribution can be used in economics. The IL distribution has many applications in modeling of different trends of hazard rate function (hrf), i.e., decreasing or upside-down bathtub failure rate of life testing of components. Many authors have studied more applications by using extended IL distribution. Recently, ZeinEldin et al. (2020) studied and introduced the alpha power extended IL (APIL) distribution. The CDF and PDF of APIL distribution with parameters α,δ and ϑ are given respectively, as

$$F(x;\alpha,\vartheta,\delta) = \frac{\alpha^{\left(1+\frac{\vartheta}{x}\right)^{-\delta}} - 1}{\alpha - 1}; \quad x > 0, \alpha, \delta, \vartheta > 0, \alpha \neq 1 \tag{1.1}$$

and

$$f(x;\alpha,\vartheta,\delta) = \frac{\ln(\alpha)}{\alpha - 1}\frac{\vartheta\delta}{x^2}\left(1 + \frac{\vartheta}{x}\right)^{-\delta-1} \alpha^{\left(1+\frac{\vartheta}{x}\right)^{-\delta}}, \alpha \neq 0. \tag{1.3}$$

Discrete Burr type XII and discrete Lomax distributions have been introduced by Para and Jan (2016). Discrete Lomax (DL) distribution is helpful in modeling discrete data which exhibits heavy tails and can be useful in medical science and other fields. The PMF and CDF of DL distribution with parameters P and ϑ are given respectively, as

$$P(x;\mathrm{P},\vartheta) = \mathrm{P}^{\ln\left(1+\frac{x}{\vartheta}\right)} - \mathrm{P}^{\ln\left(1+\frac{x+1}{\vartheta}\right)}; \quad x = \mathrm{N}_0, \vartheta > 0, 0 < \mathrm{P} < 1, \tag{1.4}$$

$$F(x;\mathrm{P},\vartheta) = 1 - \mathrm{P}^{\ln\left(1+\frac{x}{\vartheta}\right)}. \tag{1.5}$$

The question may come to mind of any researcher: why do we need discrete distributions? Since In count data analysis, we see the most of the existing continuous distributions do not set suitable results for modeling the COVID-19 cases. The cause for this as we know that counts of deaths or daily new cases show excessive dispersion.

In order to ensure members of the Australia society from the risks arising from the spread of Corona-Virus in Australia, this study aims to model the daily new cases and deaths of the COVID-19 employing a new statistical tool. An aspect of the importance of research is the necessity of mathematical and statistical modeling of the extent and spread of the Coronavirus. The discrete alpha power inverse Lomax distribution will be introduced that can be denoted as DAPIL distribution.

The rest of the paper is organized as follows. In Section 2, the survival discretization method is used. Discrete alpha power inverse Lomax distribution and maximum likelihood estimation of DAPIL are introduced in Section 3. Section 4 deals with Simulation study by using bias and MSE to inference MLE estimator. Daily new cases of COVID-19 in the case of Australia are used to validate the use of models in fitting lifetime count data are presented in Section 5. Finally, conclusions are provided in Section 6.

SURVIVAL DISCRETIZATION METHOD

In the statistics literature, sundry methods are available to obtain a discrete distribution from a continuous one. The most commonly used technique to generate discrete distribution is called a survival discretization method, it requires the existence of cumulative distribution function (CDF), survival function should be continuous and non-negative and times are divided into unit intervals. The PMF of discrete distribution is defined in Roy (2003, 2004) as follows

$$P(X=x)=P(x\leq X\leq x+1)=S(x)-S(x+1);\, x=0,1,2,\ldots, \tag{2.1}$$

Where $S(x)=P(X\geq x)=F(x;\Phi)$, where $F(x;\Phi)$ is a CDF of continuous distribution and Φ is a vector of parameters. The random variable X is said to have the discrete distribution if its CDF is given by

$$P(X<x)=F(x+1;\Phi).$$

The hazard rate is given by $hr(x)=\frac{P(X=x)}{S(x)}$. The reversed failure rate of discrete distribution is given as $rfr(x)=\frac{P(X=x)}{1-S(x)}$.

DISCRETE ALPHA POWER INVERSE LOMAX DISTRIBUTION

In this Section, we introduce a new flexible discrete model, can be donated as discrete alpha power inverse Lomax (DAPIL) distribution. Parameter estimation of DAPIL distribution is discussed by using MLE.

The DAPIL Distribution

The continuous APIL distribution is introduced by ZeinEldin et al. (2020). The survival function, of the APIL distribution, is given by

$$S(x;\alpha,\vartheta,\delta)=\frac{\alpha-\alpha^{\left(1+\frac{\vartheta}{x}\right)^{-\delta}}}{\alpha-1};\quad x>0,\gamma,\delta,\vartheta>0,$$

Using the survival discretization method and survival function of APIL distribution, we define the PMF of the DAPIL distribution as given below

$$P(x;\alpha,\vartheta,\delta)=\frac{\alpha^{\left(1+\frac{\vartheta}{x+1}\right)^{-\delta}}-\alpha^{\left(1+\frac{\vartheta}{x}\right)^{-\delta}}}{\alpha-1};x=0,1,2,\ldots \quad (3.1)$$

Let $\rho=e^{-\delta}$ then $0<\rho<1$,the PMF can be rewritten as following

$$P(x;\alpha,\vartheta,\rho)=\frac{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x}\right)}}}{\alpha-1}. \quad (3.2)$$

Figure 3 shows the PMF plots for different values of the model parameters. From Figure 3, the PMF of the DAPIL distribution is unimodal and right-skewed.

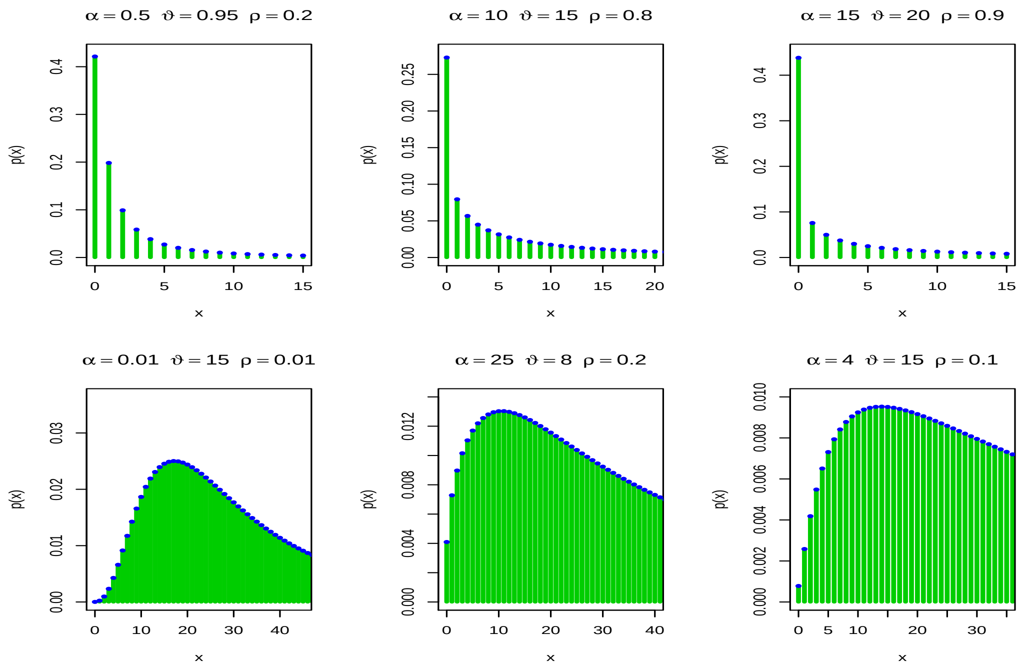

Figure 3: The PMF Plots of the DAPIL Distribution.

The CDF of the DAPIL distribution is given as below

$$F(x;\alpha,\vartheta,\rho)=\frac{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x+1}\right)}}-1}{\alpha-1}, \qquad x\in\mathbb{N}_0,$$

Where $\mathbb{N}_0=\{0,1,2,\ldots\}$. Survival function of the DAPIL distribution is given by

$$S(x;\alpha,\vartheta,\rho)=\frac{\alpha-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x+1}\right)}}}{\alpha-1}$$

The *hr* function of the DAPIL distribution is given by

$$hr(x;\alpha,\vartheta,\rho)=\frac{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x}\right)}}}{\alpha-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x+1}\right)}}},\qquad x\in\mathbb{N}_0. \tag{3.3}$$

By using Eq. (3.3), Figure 4 shows the *HRF* plots of the DAPIL distribution. It is noted that the shape of the *HRF* is increasing, left-skewed and decreasing.

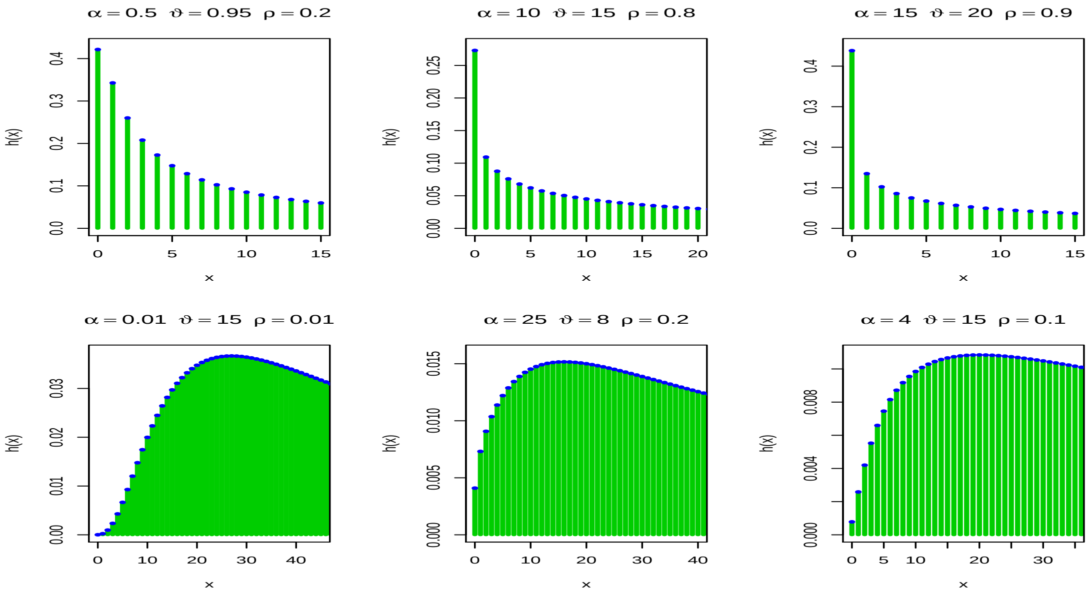

Figure 4: The HRF Plots of the DAPIL Distribution.

Parameter Estimation

The unknown parameters of the DAPIL distribution are obtained by the maximum likelihood estimation (MLE) method. This method is based on the maximization of the log-likelihood for a given data set, assume that $\boldsymbol{x}=(x_1,x_2,\ldots,x_n)^T$ is a random sample of size n from a DAPIL $(\gamma,\mathrm{P},\vartheta)$ distribution. By using Eq. (3.2), the log-likelihood function becomes

$$l(\alpha,\vartheta,\rho)=-n\ln(\alpha-1)+\sum_{i=1}^{n}\ln\left[\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}\right]. \tag{3.4}$$

Hence, the likelihood equations are

$$\frac{\partial l(\alpha,\vartheta,\rho)}{\partial\alpha}=\frac{n}{\alpha-1}+\sum_{i=1}^{n}\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}-1}-\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}-1}}{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}},$$

$$\frac{\partial l(\alpha,\vartheta,\rho)}{\partial\vartheta}=\ln(\alpha)\ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}-1}}{(x_i+1)\left(1+\frac{\vartheta}{x_i+1}\right)}-\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}{x_i\left(1+\frac{\vartheta}{x_i}\right)}}{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}},$$

and

$$\frac{\partial l(\alpha,\vartheta,\rho)}{\partial\rho}=\ln(\alpha)\sum_{i=1}^{n}\frac{\ln\left(1+\frac{\vartheta}{x_i+1}\right)\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\ln\left(1+\frac{\vartheta}{x_i}\right)\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}.$$

The estimate of the parameter by using MLE can be obtained by a numerical analysis such as the Newton–Raphson algorithm.

Asymptotic Confidence Intervals

In this subsection, we propose the asymptotic confidence intervals (CI) using MLE method to construct the confidence intervals for the parameters. First, we obtain $I(\hat{\alpha},\hat{\vartheta},\hat{\rho})$ which is the observed inverse Fishers information matrix and it is defined as:

$$I(\hat{\alpha},\hat{\vartheta},\hat{\rho})=-\begin{bmatrix}\frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\alpha^2} & & \\ \frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\alpha\partial\vartheta} & \frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\vartheta^2} & \\ \frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\alpha\partial\rho} & \frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\vartheta\partial\rho} & \frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\rho^2}\end{bmatrix} \tag{3.5}$$

See second derivatives of the logarithm of the likelihood function as follows**:**

$$\frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\alpha^2}=\frac{-n}{(\alpha-1)^2}+\sum_{i=1}^{n}\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}\left(\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}-1\right)\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}-2}-\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}\left(\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}-1\right)\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}-2}}{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}$$
$$-\sum_{i=1}^{n}\frac{\left(\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}-1}-\rho^{\ln\left(1+\frac{\vartheta}{x}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}-1}\right)^2}{\left(\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x}\right)}}\right)^2},$$

$$\frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\vartheta^2}=$$

$$(\ln(\alpha)\ln(\rho))^2\sum_{i=1}^{n}\frac{\left(\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}-1}}{(x_i+1)\left(1+\frac{\vartheta}{x_i+1}\right)}-\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}{x\left(1+\frac{\vartheta}{x_i}\right)}\right)^2}{\left(\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}\right)^2}-\ln(\alpha)\ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}-1}}{(x_i+1)^2\left(1+\frac{\vartheta}{x_i+1}\right)^2}}{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x}\right)}}}\Bigg[1+$$
$$\ln(\alpha)\ln(\rho)\,\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}+\ln(\rho)\Bigg]+\ln(\alpha)\ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}-1}}{x_i^2\left(1+\frac{\vartheta}{x_i}\right)^2}}{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x}\right)}}}\left[1-\ln(\alpha)\ln(\rho)\,\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}-\ln(\rho)\right]$$

,

$$\frac{\partial^2 l(\alpha,\theta,\rho)}{\partial\rho^2} =$$

$$\ln(\alpha)\sum_{i=1}^{n}\frac{\ln\left(1-\frac{\theta}{x_i+1}\right)\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)-2}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}\left[\ln(\alpha)\ln\left(1+\frac{\theta}{x_i+1}\right)\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}+\left(\ln\left(1+\frac{\theta}{x_i+1}\right)-1\right)\right]}{\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}}-$$

$$\ln(\alpha)\sum_{i=1}^{n}\frac{\ln\left(1-\frac{\theta}{x_i}\right)\rho^{\ln\left(1+\frac{\theta}{x_i}\right)-2}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}\left[\ln(\alpha)\ln\left(1+\frac{\theta}{x_i}\right)\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}+\left(\ln\left(1+\frac{\theta}{x_i}\right)-1\right)\right]}{\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}}-$$

$$[\ln(\alpha)]^2\sum_{i=1}^{n}\frac{\left(\ln\left(1+\frac{\theta}{x_i+1}\right)\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\ln\left(1+\frac{\theta}{x_i}\right)\rho^{\ln\left(1+\frac{\theta}{x_i}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}\right)^2}{\left(\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}\right)^2}$$

,

$$\frac{\partial^2 l(\alpha,\theta,\rho)}{\partial\alpha\partial\theta} = \ln(\alpha)\ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}}{(x_i+1)\left(1+\frac{\theta}{x_i+1}\right)}-\frac{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}}{x_i\left(1+\frac{\theta}{x_i}\right)}}{\left(\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}\right)^2}\left[\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}-1}-\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}-1}\right]+$$

$$\ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}-1}}{(x_i+1)\left(1+\frac{\theta}{x_i+1}\right)}}{\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}}\left[1+\ln(\alpha)\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}+\ln(\rho)\right]-$$

$$\ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}-1}}{x_i\left(1+\frac{\theta}{x_i}\right)}}{\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}}\left[1+\ln(\alpha)\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}+\ln(\rho)\right]$$

,

$$\frac{\partial^2 l(\alpha,\theta,\rho)}{\partial\alpha\partial\rho}$$

$$=\ln(\alpha)\sum_{i=1}^{n}\frac{\ln\left(1+\frac{\theta}{x_i+1}\right)\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\ln\left(1+\frac{\theta}{x_i}\right)\rho^{\ln\left(1+\frac{\theta}{x_i}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}}{\left(\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}\right)^2}\left[\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}-1}\right.$$

$$\left.-\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}-1}\right]+\sum_{i=1}^{n}\frac{\ln\left(1+\frac{\theta}{x_i+1}\right)\rho^{\ln\left(1-\frac{\theta}{x_i+1}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}-1}}{\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}}\left[1+\ln(\alpha)\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}\right]$$

$$-\sum_{i=1}^{n}\frac{\ln\left(1+\frac{\theta}{x_i}\right)\rho^{\ln\left(1+\frac{\theta}{x_i}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}-1}}{\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}}}\left[1+\ln(\alpha)\rho^{\ln\left(1+\frac{\theta}{x_i}\right)}\right].$$

and

$$\frac{\partial^2 l(\alpha,\vartheta,\rho)}{\partial\vartheta\partial\rho} = \ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}}{(x_i+1)\left(1+\frac{\vartheta}{x_i+1}\right)}}{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}\left[1+\ln(\alpha)\ln\left(1+\frac{\vartheta}{x_i+1}\right)+\ln\left(1+\frac{\vartheta}{x_i+1}\right)\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}\ln(\alpha)^2\right]$$

$$-\ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}{x_i\left(1+\frac{\vartheta}{x_i}\right)}}{\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}\left[1+\ln(\alpha)\ln\left(1+\frac{\vartheta}{x_i}\right)+\ln\left(1+\frac{\vartheta}{x_i}\right)\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}\ln(\alpha)^2\right]$$

$$-\ln(\alpha)^2\ln(\rho)\sum_{i=1}^{n}\frac{\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}}{(x_i+1)\left(1+\frac{\vartheta}{x_i+1}\right)}-\frac{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}}{x_i\left(1+\frac{\vartheta}{x_i}\right)}}{\left(\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}-\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}\right)^2}\left[\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i+1}\right)}}\ln\left(1+\frac{\vartheta}{x_i+1}\right)\right.$$

$$\left.-\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)-1}\alpha^{\rho^{\ln\left(1+\frac{\vartheta}{x_i}\right)}}\ln\left(1+\frac{\vartheta}{x_i}\right)\right].$$

Suppose the asymptotic variance-covariance matrix of the parameters α,ϑ,ρ is

$$V(\hat{\alpha},\hat{\vartheta},\hat{\rho}) = I^{-1}(\hat{\alpha},\hat{\vartheta},\hat{\rho}) = |I_{\hat{\alpha}\hat{\vartheta}} \quad I_{\hat{\vartheta}\hat{\vartheta}} \qquad |$$

For interval estimation, a $100(1-\gamma)\%$ confidence interval for parameter α,ϑ,ρ can be constructed based on the asymptotic normality of the MLE.

$$\hat{\alpha} \pm Z_{0.025}\sqrt{I_{\hat{\alpha}\hat{\alpha}}}, \qquad \hat{\vartheta} \pm Z_{0.025}\sqrt{I_{\hat{\vartheta}\hat{\vartheta}}};\ i=1,2, \qquad \text{and} \qquad \hat{\rho} \pm Z_{0.025}\sqrt{I_{\hat{\rho}\hat{\rho}}}$$

SIMULATION STUDY

A simulation study is assumed to evaluate and compare the behavior of the estimates with respect to their bias and mean square error (MSE). We generate 10000 random sample $x_1, x_2, \ldots,$ of sizes, n = 50, 100, 150 and 200 from DAPIL distribution. Different sets of parameters are obtained.

The MLE of α and are computed. Then, the bias and MSE of the estimates of the unknown parameters are computed. Simulated outcomes are listed in Tables 1-2 and the following observations are detected.

- The bias and MSE decrease as sample sizes increase for all estimates (see Tables 1-2).
- The bias and MSE of MLE for α,ϑ estimate is increasing with increased and and fixed ,
- The bias and MSE of MLE for α,ρ estimates are decreasing with increased ϑ and fixed ρ, α.

Table 1: Bias, MSE and CI of Parameters of DAPIL Distribution

Case	ρ		0.15				0.45				0.8			
	n		Bias	MSE	L.CI	U.CI	Bias	MSE	L.CI	U.CI	Bias	MSE	L.CI	U.CI
$\alpha = 0.5, \vartheta = 1.5$	50	α	-0.2931	0.1280	0.1956	0.6094	-0.3030	0.1641	0.3303	0.7242	0.4095	0.2351	0.4462	1.0435
		ϑ	-0.1232	0.0374	1.0845	1.6690	-0.1502	0.1274	0.7150	1.9847	-0.5124	0.6113	0.1704	2.1456
		ρ	-0.1118	0.0134	0.0203	0.0968	-0.2763	0.0875	0.0332	0.3806	-0.1310	0.0931	0.1288	1.2092
	100	α	-0.3118	0.1120	0.0504	0.4268	-0.3418	0.1382	0.1284	0.4449	0.3882	0.2155	0.4366	1.0398
		ϑ	-0.1309	0.0286	1.1594	1.5788	-0.1918	0.0900	0.8558	1.7607	-0.4749	0.5298	0.4001	1.8502
		ρ	-0.1140	0.0133	0.0012	0.0708	-0.2889	0.0880	0.0291	0.2931	-0.2114	0.1232	0.0390	1.1382
	150	α	-0.3166	0.1099	0.0092	0.3760	-0.3532	0.1378	0.0773	0.3710	0.3276	0.2036	0.4433	1.0319
		ϑ	-0.1340	0.0251	1.2002	1.5317	-0.2039	0.0908	0.8608	1.7313	-0.4060	0.5038	0.3139	1.7019
		ρ	-0.1138	0.0131	0.0095	0.0630	-0.2902	0.0876	0.0464	0.2732	-0.1931	0.0948	0.1364	1.0774
	200	α	-0.3160	0.1068	0.0202	0.3478	-0.3579	0.1375	0.0480	0.3323	0.2893	0.1918	0.4569	1.0218
		ϑ	-0.1266	0.0232	1.2075	1.5393	-0.1874	0.0779	0.9068	1.7185	-0.3527	0.4276	0.3465	1.7412
		ρ	-0.1136	0.0131	0.0123	0.0604	-0.2892	0.0863	0.0588	0.2628	-0.2302	0.1293	0.0281	1.1115
$\alpha = 0.25, \vartheta = 1.75$	50	α	-0.1431	0.0339	0.1205	0.3342	-0.1245	0.0514	0.2459	0.4969	0.1799	0.1483	0.2379	1.0977
		ϑ	-0.0892	0.0398	1.3108	2.0107	-0.2087	0.1417	0.9271	2.1556	-0.4150	0.3810	0.4391	2.2308
		ρ	-0.1039	0.0115	-0.0041	0.0962	-0.2533	0.0748	0.0054	0.3988	-0.1942	0.1149	0.0611	1.1505
	100	α	-0.1532	0.0280	0.0347	0.2282	-0.1556	0.0345	0.1046	0.2934	0.1794	0.1221	0.1585	1.0173
		ϑ	-0.1061	0.0416	1.3026	1.9852	-0.1951	0.1053	1.0464	2.0633	-0.4786	0.4555	0.3382	2.2046
		ρ	-0.1061	0.0115	0.0113	0.0766	-0.2610	0.0722	0.0637	0.3143	-0.1841	0.0953	0.1298	1.1020
	150	α	-0.1563	0.0276	0.0166	0.2041	-0.1626	0.0316	0.0531	0.2279	0.1245	0.1009	0.1984	0.9474
		ϑ	-0.1118	0.0398	1.3142	1.9622	-0.2389	0.1325	0.9725	2.0497	-0.5554	0.4728	0.3996	1.9895
		ρ	-0.1055	0.0113	0.0162	0.0728	-0.2653	0.0736	0.0742	0.2952	-0.2090	0.1010	0.1213	1.0606
	200	α	-0.1561	0.0262	0.0100	0.1778	-0.1658	0.0309	0.0311	0.1996	0.0224	0.0751	0.2632	0.8080
		ϑ	-0.1229	0.0442	1.2924	1.9618	-0.2205	0.1033	1.0708	1.9882	-0.7133	0.6699	0.2494	1.8241
		ρ	-0.1063	0.0115	0.0161	0.0714	-0.2638	0.0722	0.0855	0.2868	-0.2935	0.1411	0.0467	0.9662

Table 2: Bias, MSE and CI of Parameters of DAPIL Distribution

Case	ρ		0.15				0.45				0.8			
	n		Bias	MSE	L.CI	U.CI	Bias	MSE	L.CI	U.CI	Bias	MSE	L.CI	U.CI
$\alpha = 2.5, \vartheta = 2$	50	α	-0.5942	0.4177	1.4074	2.4041	-0.7021	0.5689	1.2571	2.3386	-0.3976	0.1664	1.9234	2.2815
		ϑ	-0.3560	0.3319	0.7558	2.5321	-1.0408	1.6432	0.5096	2.4280	-0.7379	1.1910	0.3146	2.8387
		ρ	-0.0783	0.1590	0.0695	0.2684	-0.3275	0.1247	0.1365	0.3814	-0.2744	0.1465	0.0023	1.0490
	100	α	-0.5063	0.2824	1.6770	2.3104	-0.6268	0.4270	1.5110	2.2354	-0.4048	0.1707	1.9332	2.2572
		ϑ	-0.2061	0.1274	1.2224	2.3655	-0.8039	0.9245	0.1617	2.2305	-0.8256	1.0707	0.2572	2.3975
		ρ	-0.0781	0.0066	0.0968	0.2270	-0.2683	0.0859	0.0494	0.4128	-0.2461	0.0943	0.1939	0.9138
	150	α	-0.4839	0.2577	1.7152	2.3170	-0.7239	0.5434	1.5025	2.0497	-0.4234	0.1853	1.9241	2.2291
		ϑ	-0.1635	0.0877	1.3525	2.3206	-1.0822	1.3317	0.1320	1.7037	-0.7860	0.9405	0.2760	2.3040
		ρ	-0.0698	0.0056	0.0969	0.1942	-0.3299	0.1153	0.0582	0.2784	-0.3054	0.1423	0.0604	0.9288
	200	α	-0.6148	0.4078	1.5468	2.2236	-0.6817	0.4822	1.5592	2.0773	-0.4300	0.1893	1.9398	2.2001
		ϑ	-0.3878	0.2337	1.0463	2.1782	-0.9575	1.0609	0.2982	1.7867	-0.7080	0.9057	0.3231	2.1625
		ρ	-0.0929	0.0095	0.1092	0.1764	-0.3010	0.0978	0.0617	0.3155	-0.3414	0.1699	0.0057	0.9116
$\alpha = 2.5, \vartheta = 3$	50	α	-0.3413	0.2433	1.4596	2.8578	-0.5491	0.4691	1.1477	2.7541	-0.0754	0.0240	2.1592	2.6900
		ϑ	-0.5070	0.5219	1.4828	3.5031	-0.9275	1.1611	0.2228	3.6733	-0.5626	1.2090	0.5848	4.2900
		ρ	-0.0719	0.0070	0.0061	0.1623	-0.2608	0.0830	0.0507	0.4290	-0.1485	0.0706	0.2195	1.0836
	100	α	-0.2696	0.1424	1.7128	2.7479	-0.3497	0.1947	1.6227	2.6779	-0.1371	0.0419	2.0646	2.6613
		ϑ	-0.3965	0.3109	1.8348	3.3723	-0.8067	0.9854	1.0589	3.3277	-0.5037	0.9949	0.2385	3.6878
		ρ	-0.0660	0.0052	0.0273	0.1407	-0.1787	0.0417	0.0779	0.4647	-0.1746	0.0552	0.3173	0.9336
	150	α	-0.1291	0.0355	2.1015	2.6403	-0.5455	0.4004	1.3255	2.5835	-0.2115	0.0658	2.0038	2.5731
		ϑ	-0.1928	0.0796	2.4034	3.2109	-0.7309	0.9010	0.6241	2.7575	-0.4626	0.9146	0.4072	3.1619
		ρ	-0.0486	0.0027	0.0652	0.1376	-0.2553	0.0715	0.0389	0.3505	-0.2869	0.1252	0.1068	0.9194
	200	α	-0.2477	0.1230	1.7656	2.7389	-0.4501	0.2667	1.5532	2.5466	-0.1548	0.0372	2.1193	2.5711
		ϑ	-0.3673	0.2493	1.9694	3.2959	-0.6032	0.8380	0.8660	3.0707	-0.4350	0.8392	0.6078	3.1507
		ρ	-0.0617	0.0050	0.0216	0.1550	-0.2129	0.0544	0.0504	0.4238	-0.2209	0.0857	0.2026	0.9556

APPLICATION ANALYSIS

In this section, the DAPITL distribution is fitted to more famous fields of survival times of Covid-19 in Australia. We compare the fits of the discrete Marshall-Olkin generalized exponential (DMOGEx) [Almetwally et al. (2020)] model, discrete generalized exponential (DGEx) [Nekoukhou et al. (2013)], natural discrete Lindley (NDL) [Al-Babtain et al. (2020)], discrete Gompertz exponential (DGzEx) [El-Morshedy et al. (2020)], discrete Burr (DB) [Krishna and Pundir (2009)], discrete Lindley (DLi) [Gómez-Déniz and Calderín-Ojeda (2011)], discrete Lomax (DLo) [Para and Jan (2016)] and Geometric models in Tables 3.

Table 3, provide values of AIC, CAIC, BIC, HQIC and Kolmogorov- Smirnov (KS) statistic along with its P-value for the all models fitted based on real data set of Australia. In addition, this table contains the MLE and standard errors (SE) of the parameters for the considered models. The fitted DAPIL, PMF, CDF, PP-plot and QQ-plot of the data sets of Australia are displayed in Figure 5. These figures indicate that the DAPIL distribution get the lowest values of AIC, CAIC, BIC, HQIC, KS and largest P-value, among all fitted models.

This is a COVID-19 data belong to Australia of 32 days, that is recorded from 3September to 4October 2020. This data formed of daily new cases. The data are as follows:6, 15, 59,11, 5, 9, 8,11, 7, 9,6,7, 6, 0, 8, 8, 5, 7, 5, 2, 3,5, 2, 8, 1, 2, 3, 7, 4, 2, 2, 3.

Table 3: ML Estimates, K-S, P-Values, AIC, CAIC, BIC and HQIC for COVID-19 Data in Australia Data

		Estimate	SE	KS	P-Value	AIC	CAIC	BIC	HQIC
DAPILo	α	0.0030	0.0039	0.1533	0.4393	190.4229	191.2801	194.8201	191.8805
	ϑ	2.9569	1.4151						
	ρ	0.0074	0.0121						
DB	ϑ	4.2572	2.7233	0.4080	0.0000	218.2952	218.7090	221.2267	219.2669
	ρ	0.8731	0.0781						
Dli	ρ	0.7943	0.0231	0.1620	0.3704	196.2441	196.3774	197.7098	196.7300
DGE	α	0.8287	0.0332	0.1442	0.5190	195.3791	195.7928	198.3105	196.3508
	ϑ	1.8918	0.5451						
DMOGE	α	2.6244	0.4757	0.1885	0.2056	190.8208	191.6780	195.2180	192.2784
	ϑ	0.0075	0.0089						
	ρ	0.0294	0.0190						
Geom	ρ	0.1194	0.0198	0.2546	0.0315	198.0333	198.1666	199.4990	198.5192
Dlo	ϑ	28.3501	16.2203	0.2015	0.1487	200.1520	200.5658	203.0834	201.1237
	ρ	0.0111	0.0253						
NDL	ρ	0.1991	0.0226	0.2382	0.0530	196.5304	196.6637	197.9961	197.0162

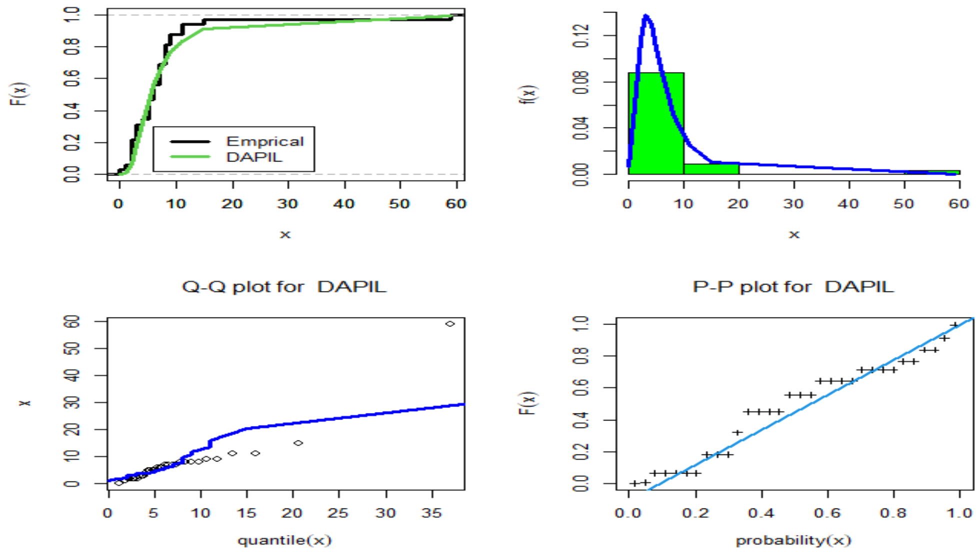

Figure 5: Estimated PMF, CDF, PP-Plot and QQ-Plot of DAPIL for COVID-19 data in Australia Data.

CONCLUDING REMARKS

In this article, with the aim of managing the risk of spreading Coronavirus in Australia, we proposed and studied the discrete Marshall–Olkin Lomax distribution. The maximum likelihood estimation method is discussed to estimate the parameter of DAPIL distribution. Monte Carlo Simulations are obtained to evaluate the restricted sample properties of the DAPIL distribution. We prove empirically that the DAPIL model reveals its superiority over other competitive models as Marshall-Olkin generalized exponential, discrete generalized exponential, natural discrete Lindley, discrete Gompertz exponential, discrete Burr, discrete Lindley, discrete Lomax and Geometric for analysis daily new cases of the COVID-19 in the case of Australia. These figures indicate that the DAPIL distribution gets the lowest values of AIC, CAIC, BIC, HQIC, KS, and the largest P-value, among all fitted models.

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